

Nonstatic global string in Brans-Dicke theory

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Abstract

Gravitational field of a nonstatic global string has been studied in the context of Brans-Dicke theory of gravity. Both the metric components and the BD scalar field are assumed to be nonseparable functions of time and space. The spacetime may or may not have any singularity at a finite distance from the string core but the singularity at a particular time always remains. It has been shown that the spacetime exhibits both outgoing and incoming gravitational radiation.

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I. INTRODUCTION

In the recent years there has been a rapid growth of research in the interface between particle physics and cosmology resulting in several exciting ideas concerning the unification of forces of nature. One of the most important features in the early universe, particle physicists and cosmologists have speculated, is the "Grand

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Unification”- unification of the strong and electroweak forces. It was soon realised that this grand symmetry presented in the early universe at very high energy scale had to be broken in order to account for the electroweak and nuclear forces as different fields in the present day low energy universe. One of the immediate and inescapable consequences of this symmetry breaking is the formation of topological defects. They can be monopoles, cosmic strings or domain walls [1].

Among these defects, cosmic strings are particularly interesting in view of their capability to produce direct observational effects such as gravitational lensing and also because they are possible seeds of galaxy formation [2]. Strings are said to be local or global depending on their origin from the breakdown of local or global $U(1)$ symmetry. While local strings are well behaved [3], having an exterior representing a flat Minkowskian spacetime with a conical defect, global strings, however, have strong gravitational effects at large distances. The static global string spacetime is found to have a singularity at a finite distance [4] from the string core. Later Gregory [5] showed that the time dependence of the metric might remove the singular behaviour of the global string spacetime. Very recently Sen and Banerjee have shown that one can also have nonsingular static spacetime outside the core of the global string if the spacetime does not admit Lorentz boost along the symmetry axis [6].

Discussions in the previous paragraphs have been confined within the context of General Relativity (GR). But at sufficiently high energy scales, it seems likely that gravity is not given by the Einstein action but becomes modified by the superstring terms. In the low energy limits of this string theory, one recovers Einstein’s gravity along with scalar dilaton field which is nonminimally coupled to the gravity [7]. On the other hand, scalar tensor theories, such as Brans-Dicke theory (BD) [8], which is compatible with the Mach’s principle, have been considerably revived in the recent years. In these theories, the purely metric coupling of matter with gravity is restored and hence the equivalence principle is ensured. Moreover these models exhibit an attractor mechanism towards GR, so that the expansion of the universe during matter dominated epoch tends to drive the scalar field towards a state where these models are indistinguishable from GR [9]. Although dilaton gravity and BD theory arise from entirely different motivations, it can be shown that the former is special case of the later, at least formally [10]. The implications of the BD theory for topological defects have been studied by many authors. Gundlach and Ortiz [11] obtained analytical

solutions for a local gauge string in BD theory. These solutions are not satisfactory because here the BD theory was used to solve for the exterior, while in the interior, the solutions were given for Einstein's field equations, ignoring the BD scalar field completely. The other solutions in BD theory for a local gauge string have been given by Barros and Romero [12] and Guimaraes [13]. Later Sen et.al [14] have shown that for a local gauge string, Vilenkin's prescription for the energy momentum tensor due to the string is inconsistent with BD theory. Gravitational field of a $U(1)$ global string has also been studied by many authors. Sen et.al [14] have presented two kind of solutions for the spacetime outside the core of a static global string, one of which was in closed form and was nonsingular at a finite distance from the string core while the other one could not be written in a closed form and moreover contained singularity at a finite distance from the string core. More general solutions for static global string in BD theory have been provided by Boisseau and Linet [15]. In another interesting work, Dando and Gregory [16] have examined field equations of a self gravitating nonstatic global string in low energy string gravity allowing for an arbitrary coupling of the global string to the dilaton field. Both massive and massless dilaton fields were considered. In both cases, they had demonstrated the existence of a nonsingular spacetime for the string if one includes a time dependence of the form $e^{b_0 t}$ along the length of the string where $b_0 > 0$. But here the dilaton scalar field itself was not time dependent. In this work, we have studied the gravitational field of a nonstatic global string in BD theory where the BD scalar field is also time dependent along with the metric components. Moreover the metric components are not separable functions of time and space which was the assumption for most of the previous works.

In section II, we have constructed the field equations for the time dependent global string in BD theory and have presented the solutions for the field equations. In section III, we have analysed our solutions where we have studied the presence of the physical singularity in our spacetime and also have shown that the spacetime represents both incoming and outgoing gravitational radiations. The paper ends with a conclusion in section IV.

II. FIELD EQUATIONS AND THEIR SOLUTIONS

The gravitational field equation in BD theory is given by [8]

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{\Phi} + \frac{\omega}{\Phi^2}(\Phi_{,\mu}\Phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\Phi_{,\alpha}\Phi^{,\alpha}) + \frac{1}{\Phi}(\Phi_{,\mu;\nu} - g_{\mu\nu}\square\Phi) \quad (1.1),$$

together with the wave equation for the BD scalar field Φ

$$\square\Phi = \frac{T}{(2\omega + 3)} \quad (1.2),$$

where Φ is the BD scalar field, $T_{\mu\nu}$ is the energy momentum tensor for the matter field and ω is the BD parameter. The conservation relation for the matter field $T_{;\nu}^{\mu\nu} = 0$ follows identically. Sometimes it is more convenient to use a nonphysical metric $\bar{g}_{\mu\nu}$ which is conformally related to $g_{\mu\nu}$ by [17]

$$g_{\mu\nu} = \Phi^{-1}\bar{g}_{\mu\nu} \quad (1.3),$$

and the corresponding field equations are

$$\bar{G}_{\mu\nu} = \bar{T}_{\mu\nu} + \frac{(2\omega + 3)}{2\Phi^2}(\Phi_{,\mu}\Phi_{,\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\Phi^{,\alpha}\Phi_{,\alpha}) \quad (1.4),$$

$$\square\Phi = \frac{\bar{T}}{2\omega + 3} \quad (1.5),$$

where \square operator is calculated with respect to $\bar{g}_{\mu\nu}$ and also now $\bar{T}_{\mu\nu}$ is the conformally transformed energy momentum tensor given by

$$T_{\mu\nu} = \Phi\bar{T}_{\mu\nu} \quad (1.6).$$

But now the conservation equation for the matter field does not follow identically. The rest mass of the particles vary depending on the spacetime locations and hence the paths of the massive particles are no longer geodesics.

We have taken the line element as

$$ds^2 = e^{2(k-u)}(dt^2 - dr^2) - e^{2u}dz^2 - w^2e^{-2u}d\theta^2 \quad (1.7),$$

where k, u, w are functions of both t and r . Assuming that the complex scalar field for the global string is independent of the radial distance r outside the core radius $\delta = (\eta\sqrt{\lambda})^{-1}$, where η is the symmetry breaking energy scale and λ is a constant,

one can calculate the energy momentum tensor for the global string in the original version of BD theory as

$$T_t^t = T_r^r = T_z^z = -T_\theta^\theta = \frac{\eta^2}{2} \frac{e^{2u}}{w^2} \quad (1.8).$$

In our case the T_ν^μ s are conformally transformed as (1.6), hence we assume \bar{T}_ν^μ in the conformally transformed version as

$$\bar{T}_t^t = \bar{T}_r^r = \bar{T}_z^z = -\bar{T}_\theta^\theta = \bar{\sigma}(r, t) \quad (1.9)$$

With (1.7) and (1.9), equations (1.4) and (1.5) become

$$\frac{\dot{k}\dot{w}}{w} - \dot{u}^2 - \frac{w''}{w} + \frac{k'w'}{w} - u'^2 = -\bar{\sigma}e^{2(k-u)} - \frac{(2\omega+3)}{4}(\dot{\psi}^2 + \psi'^2) \quad (1.91a)$$

$$\frac{\ddot{w}}{w} - \frac{\dot{k}\dot{w}}{w} + \dot{u}^2 - \frac{k'w'}{w} + u'^2 = -\bar{\sigma}e^{2(k-u)} + \frac{(2\omega+3)}{4}(\dot{\psi}^2 + \psi'^2) \quad (1.91b)$$

$$\dot{u}^2 + \ddot{k} - u'^2 - k'' = \bar{\sigma}e^{2(k-u)} - \frac{(2\omega+3)}{4}(\dot{\psi}^2 - \psi'^2) \quad (1.91c)$$

$$\frac{\ddot{w}}{w} - \frac{2\dot{u}\dot{w}}{w} - 2\ddot{u} + \dot{u}^2 + \ddot{k} - \frac{w''}{w} + \frac{2u'w'}{w} + 2u'' - k'' - u'^2 = -\bar{\sigma}e^{2(k-u)} + \frac{(2\omega+3)}{4}(\dot{\psi}^2 - \psi'^2) \quad (1.91d)$$

$$\frac{k'\dot{w}}{w} - \frac{\dot{w}'}{w} + \frac{\dot{k}w'}{w} - 2u'\dot{u} = -\frac{(2\omega+3)}{2}\dot{\psi}\psi' \quad (1.91e)$$

$$\ddot{\psi} + \frac{\dot{\psi}\dot{w}}{w} - \psi' - \frac{\psi'w'}{w} = \frac{2\bar{\sigma}e^{2(k-u)}}{(2\omega+3)} \quad (1.91f)$$

where we have assumed $\psi = \ln(\Phi)$ and overdot and prime represent differentiation with respect to t and r respectively. After some straightforward calculations one can get

$$(\dot{u}w)' = (u'w)' \quad (1.92).$$

One of the possible solutions of (1.92) is that both $(\dot{u}w)$ and $(u'w)$ are functions of $(r+t)$. In what follows, we have assumed that k, u, w, ψ, σ are functions of $(at+br)$ in our subsequent calculations where a, b are arbitrary constants.

Defining $x = at + br$, equations (1.91a)-(1.91f) become

$$(a^2 + b^2)\frac{k_x w_x}{w} - (a^2 + b^2)u_x^2 - b^2\frac{w_{xx}}{w} = -\bar{\sigma}e^{2(k-u)} - \frac{(2\omega+3)}{4}(a^2 + b^2)\psi_x^2 \quad (1.93a)$$

$$a^2\frac{w_{xx}}{w} - (a^2 + b^2)\frac{k_x w_x}{w} + (a^2 + b^2)u_x^2 = -\bar{\sigma}e^{2(k-u)} + \frac{(2\omega+3)}{4}(a^2 + b^2)\psi_x^2 \quad (1.93b)$$

$$(a^2 - b^2)u_x^2 + (a^2 - b^2)k_{xx} = \bar{\sigma}e^{2(k-u)} + \frac{(2\omega + 3)}{4}(a^2 - b^2)\psi_x^2 \quad (1.93c)$$

$$\begin{aligned} (a^2 - b^2)\frac{w_{xx}}{w} - 2(a^2 - b^2)\frac{u_x w_x}{w} - 2(a^2 - b^2)u_{xx} + (a^2 - b^2)u_x^2 + (a^2 - b^2)k_{xx} \\ = -\bar{\sigma}e^{2(k-u)} + \frac{(2\omega + 3)}{4}(a^2 - b^2)\psi_x^2 \end{aligned} \quad (1.93d)$$

$$(a^2 - b^2)(\psi_{xx} + \frac{\psi_x w_x}{w}) = \frac{2\bar{\sigma}e^{2(k-u)}}{2\omega + 3} \quad (1.93e)$$

It can be shown that equation (1.91e) is now no longer independent but can be achieved with the help of other equations. After some straightforward calculations one can get from (1.93a)-(1.93e) the following equations:

$$(a^2 - b^2)\frac{w_{xx}}{w} = -2\bar{\sigma}e^{2(k-u)} \quad (1.94a)$$

$$\frac{w_{xx}}{w} - \frac{2k_x w_x}{w} + 2u_x^2 = \frac{(2\omega + 3)}{2}\psi_x^2 \quad (1.94b)$$

$$u_x = \alpha/w \quad (1.94c)$$

$$k_x = \beta/w \quad (1.94d)$$

$$e^\psi = Bw^{-1/(2\omega+3)} \quad (1.94e)$$

where α , β and B are arbitrary integration constants. So we have five equations and five unknowns namely, w , k , u , ψ and $\bar{\sigma}$. Also one should note from equation (1.94a) that $a^2 \neq b^2$ in order to have nonvanishing $\bar{\sigma}$. Using (1.94c)-(1.94e), (1.94b) becomes

$$ww_{xx} - 2\beta w_x + 2\alpha^2 - \frac{1}{2(2\omega + 3)}w_x^2 = 0 \quad (1.95).$$

To have an exact analytical solution for the equation (1.95), we make a simplified assumption that $\alpha = \beta = 0$ which means

$$u_x = k_x = 0 \quad (1.96).$$

Then one can integrate equation (1.95) to get

$$w = w_0(x - x_0)^{1/(1-A)} \quad (1.97)$$

where w_0 and x_0 are arbitrary constants and $A = \frac{1}{2(2\omega+3)}$. One can now calculate the expressions for the energy density $\bar{\sigma}$ and the BD scalar field Φ :

$$\bar{\sigma} = \frac{p(p-1)(b^2 - a^2)}{2(x - x_0)^2} \quad (1.98a)$$

$$\Phi = e^\psi = \Phi_0(x - x_0)^{2A/(A-1)} \quad (1.98b)$$

where we have taken the constants $e^k = e^u = 1$ without any loss of generality and $p = 1/(1 - A)$. Using (1.3) and (1.6) one can now calculate the expression for the line element and the energy density in the original BD theory which are given by

$$ds^2 = \frac{1}{\Phi_0}(at + br - x_0)^{2A/(1-A)}(dt^2 - dr^2 - dz^2) - \frac{w_0^2}{\Phi_0}(at + br - x_0)^{2(1+A)/(1-A)}d\theta^2 \quad (1.99a)$$

$$\sigma = \frac{p(p-1)(b^2 - a^2)\Phi_0^2}{2}(at + br - x_0)^{-2(1+A)/(1-A)} \quad (1.99b).$$

One can check that the energy density $\sigma \propto 1/g_{33}$ which is true for global string according to (1.8). To recover the corresponding solution for Einstein's General Relativity one has to take the limit $\omega \rightarrow \infty$ [8]. In the present case, in this limit, the BD scalar field Φ becomes constant and the line element (1.99a) becomes a flat metric with an angular deficit.

One can also calculate the angular deficit in the spacetime for a constant time which becomes

$$2\pi \left[1 - \frac{bw_0(at + br - x_0)^{(1+A)/(1-A)}}{(1-A)[(at + br - x_0)^{1/(1-A)} - (at - x_0)^{1/(1-A)}]} \right].$$

Hence it is clear from the above expression that the angular deficit is a function of both r and t .

III. ANALYSIS OF THE SOLUTION

To check the occurrence of the curvature singularity in the spacetime, one has to calculate the Kretschmann curvature scalar for the line element (1.99a):

$$R^{ijkl}R_{ijkl} = 4\Phi_0^2 \left[\frac{(a^2 - b^2)^2(80\omega^2 + 248\omega + 198)(at + br - x_0)^{-8(2\omega+3)/(4\omega+7)}}{(16\omega^2 + 40\omega + 25)^2} \right] \quad (2.1)$$

One can see that for $x_0 \leq 0$, the spacetime does not have any curvature singularity at any finite distance from the string core if $(2\omega + 3) > 0$ which is a physical assumption so as to make the energy contribution from the scalar field positive [9]. There may be singularity at $r = 0$ but line element (1.99a) is valid for $r > \delta$ because the form of the energy momentum tensor given in (1.8) is valid only for outside the string core that is $r > \delta$. For $x_0 > 0$ the spacetime may have singularity at a finite distance from

the string core. But there is always a physical singularity at a particular time in the spacetime depending on the choice of the constant as the range of t is $-\infty \leq t \leq \infty$.

To check whether the spacetime represents the emission and absorption of gravitational radiation, one has to calculate the corresponding Weyl tensor, which is thought of as representing the pure gravitational field [18]. Introducing the null tetrad frame by

$$L^\mu = \frac{e^{u-k}}{\sqrt{2}}(\delta_t^\mu - \delta_r^\mu) \quad (2.2a)$$

$$N^\mu = \frac{e^{u-k}}{\sqrt{2}}(\delta_t^\mu + \delta_r^\mu) \quad (2.2b)$$

$$M^\mu = \frac{1}{\sqrt{2}}(e^{-u}\delta_z^\mu + \frac{i}{w}\delta_\theta^\mu) \quad (2.2c)$$

$$\bar{M}^\mu = \frac{1}{\sqrt{2}}(e^{-u}\delta_z^\mu - \frac{i}{w}\delta_\theta^\mu) \quad (2.2d)$$

one can find that the two of the nonvanishing components of the Weyl tensor for the line element (1.99a) are

$$\Psi_0 = -C_{\mu\nu\lambda\sigma}L^\mu M^\nu L^\lambda M^\sigma \quad (2.3a),$$

$$\Psi_4 = -C_{\mu\nu\lambda\sigma}N^\mu \bar{M}^\nu N^\lambda \bar{M}^\sigma \quad (2.3b).$$

The reason for calculating these two components in null frame is that these components in this frame have a direct physical meaning that is Ψ_0 represents an outgoing cylindrical gravitational wave along the null hypersurface defined by N^μ and Ψ_4 represents the incoming cylindrical gravitational wave propagating along the null hypersurface defined by L^μ [19]. Using the line element (1.99a) and (2.2) and (2.3) these components become

$$\begin{aligned} \Psi_0 = & \frac{3A\Phi_0^3(a^2 + b^2)(at + br - x_0)^{-2(1+2A)(1-A)}}{24(1-A)^2} \left[\frac{\Phi_0(at + br - x_0)^{-2(1+A)/(1-A)}}{w_0^2} + 1 \right] \\ & - \frac{abA\Phi_0^3(a^2 + b^2)(at + br - x_0)^{-2(1+2A)(1-A)}}{24(1-A)^2} \left[\frac{\Phi_0(at + br - x_0)^{-2(1+A)/(1-A)}}{w_0^2} + 1 \right] \end{aligned} \quad (2.4a)$$

$$\begin{aligned} \Psi_4 = & \frac{3A\Phi_0^3(a^2 + b^2)(at + br - x_0)^{-2(1+2A)(1-A)}}{24(1-A)^2} \left[\frac{\Phi_0(at + br - x_0)^{-2(1+A)/(1-A)}}{w_0^2} + 1 \right] \\ & + \frac{abA\Phi_0^3(a^2 + b^2)(at + br - x_0)^{-2(1+2A)(1-A)}}{24(1-A)^2} \left[\frac{\Phi_0(at + br - x_0)^{-2(1+A)/(1-A)}}{w_0^2} + 1 \right] \end{aligned} \quad (2.4b)$$

The first terms in the expressions for Ψ_0 and Ψ_4 are equal and are nonvanishing for $a = 0$ i.e for static spacetime. They represent the gravitational field for the global string. The second terms in Ψ_0 and Ψ_4 are of opposite sign and does not exist for $a = 0$ i.e for static spacetime. These terms represent the outgoing and incoming gravitational radiation respectively. For a particular case $ab = 3$ one can see that there is no outgoing radiation from the string but only the incoming radiation exists.

IV. CONCLUSION

In this work we have studied the gravitational field outside the core of a nonstatic global string in BD theory of gravity. The solution presented here are not the general one as we have made some assumptions but this may be the first example of such investigations where the BD scalar field is itself time dependent and also where the metric components are not the separable function of t and r . The previous work in this line by Dando and Gregory [16], assumed the metric components are separable functions of time and space and also the BD scalar field was time independent. The solution presented here may or may not have singularity at a finite distance from the string core depending upon the value of arbitrary constant but it has singularity at a particular time. One can also check from the expression of Kretschmann scalar that the spacetime is asymptotically flat both in time and space for $(2\omega + 3) > 0$ which is a physical assumption [8]. As the components of Weyl tensor in null frame are nonzero hence there are incoming as well as outgoing gravitational radiation in our spacetime. Also the fact that the weyl tensor has non zero components combined with the non zero Ricci tensor leads to the fact that these strings may produce some gravitational lens effects for null rays passing near the strings. Hence there may be distortion and amplifications of distance objects seen along the lines of sight passing near the string [20].

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